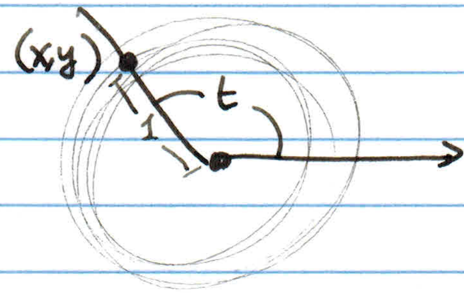
1060 - Day 16

Fall 2013

Recall sine & cosine defined using unit circle



Say

$$\sin(t) = \frac{y}{1} = y$$

$$\cos(t) = \frac{x}{1} = x$$

to <u>compute</u> $\sin(t)$

① Graph t & find reference $\measuredangle$ $\hat{t}$

② find $\sin(\hat{t})$

<u>and</u>

③ use graph of t to decide if y-coordinate is <u>pos</u> or <u>neg</u>

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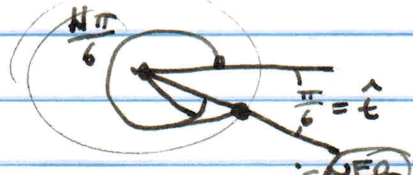
<u>Eg:</u> $\sin\left(\frac{11\pi}{6}\right)$

①

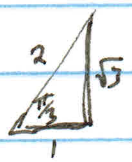


② $\sin(\hat{t}) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

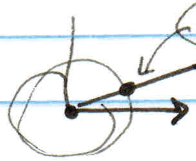
③ the y-coordinate is NEG at $\hat{t}$.

Eg: find all solutions θ in $[0, 2\pi]$

(a) $\sin(\theta) = \frac{1}{2}$



this is a solution



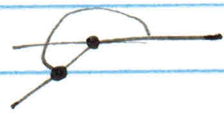
Remember: $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\Rightarrow$ $\sin(\theta) = \frac{1}{2} \Leftrightarrow$ Reference $\measuredangle = \frac{\pi}{6}$ AND y-coord is POSITIVE

What other $\measuredangle$'s are solutions?

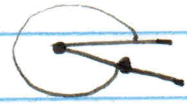
$$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

$$\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$$



} y-coord is neg

$$\sin\left(\frac{11\pi}{6}\right) = \frac{-1}{2}$$



} y-coord is neg.

(b) $\sin(\theta) = \frac{-1}{2}$

Notice $\sin(\theta) = -\frac{1}{2}$

$\Leftrightarrow$

Reference $\measuredangle$ is $\frac{\pi}{6}$

AND y-coordinate is negative

So $\sin\theta = -\frac{1}{2}$

when $\theta = \frac{7\pi}{6}$ or $\theta = \frac{11\pi}{6}$

4.5 - Graphing Sine & Cosine.

Now we'll shift from "computing $\sin(x)$"
to
"thinking of $\sin(x)$ as a function".

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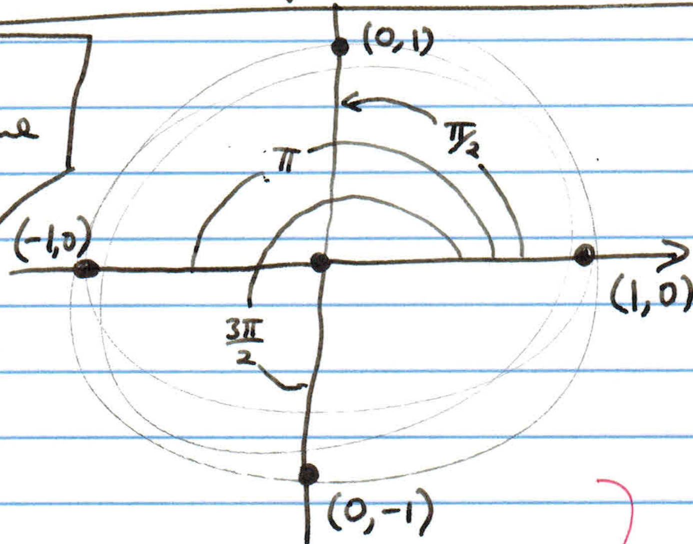
Domain of $\sin(x)$ is $(-\infty, \infty)$ ← defined for all x's t

Range of $\sin(x)$ is $[-1, 1]$ ← y-coordinate is always between -1 & 1

~~Similarly Domain of cos(~~

To graph $\sin(x)$
graph it at $x=0, \frac{\pi}{2}, \pi, \frac{3\pi}{2},$ & 2π
and fill in gaps smoothly.

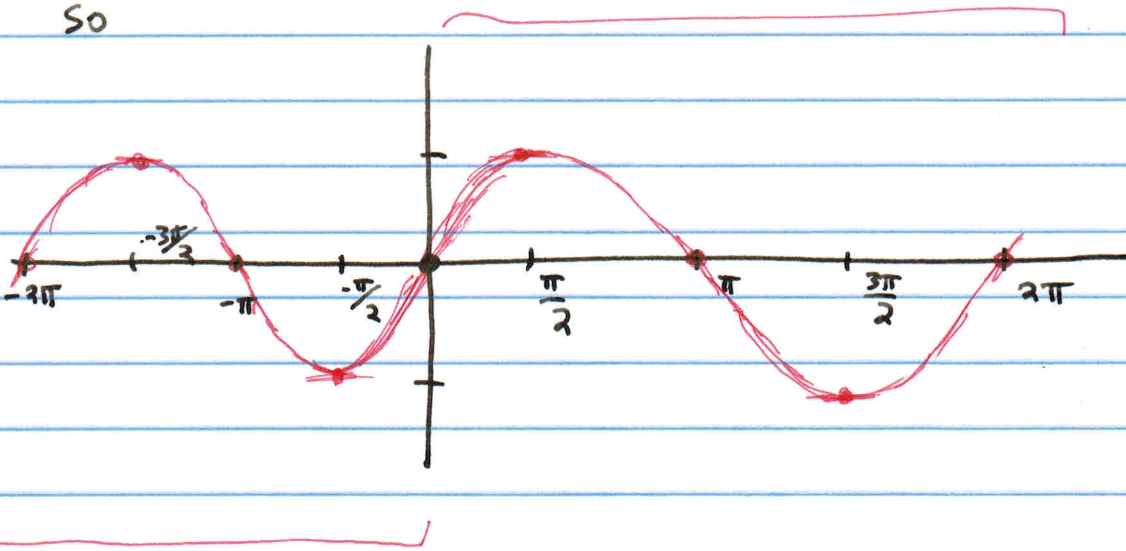
REMEMBER
$\sin(t)$ = y-value on the unit circle



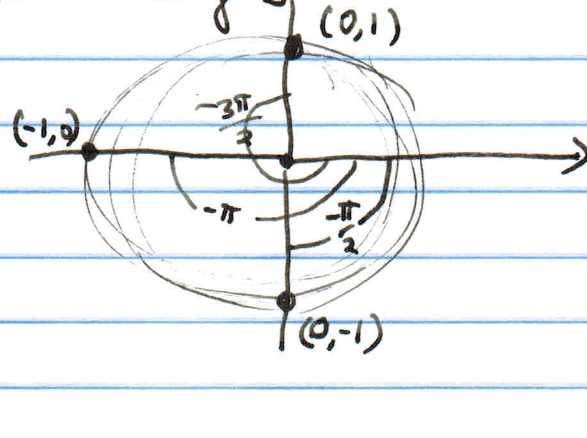
QUICK

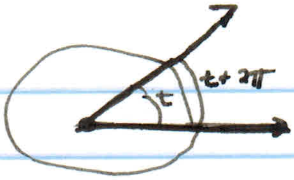
5

so



for negative angles

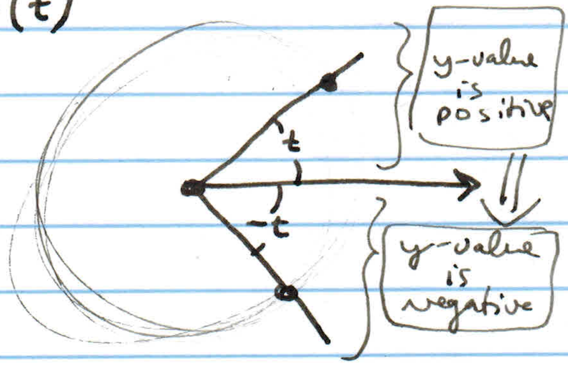




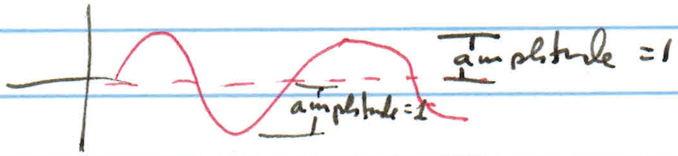
Notice: $\sin(t + 2\pi) = \sin(t)$

> So we say that $\sin(t)$
> is PERIODIC
> 5 with period $= 2\pi$

Also Notice: $\sin(-t) = -\sin(t)$



The amplitude is the amount that waves go up from their middle

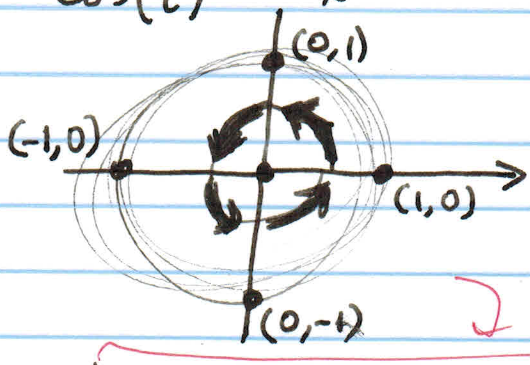


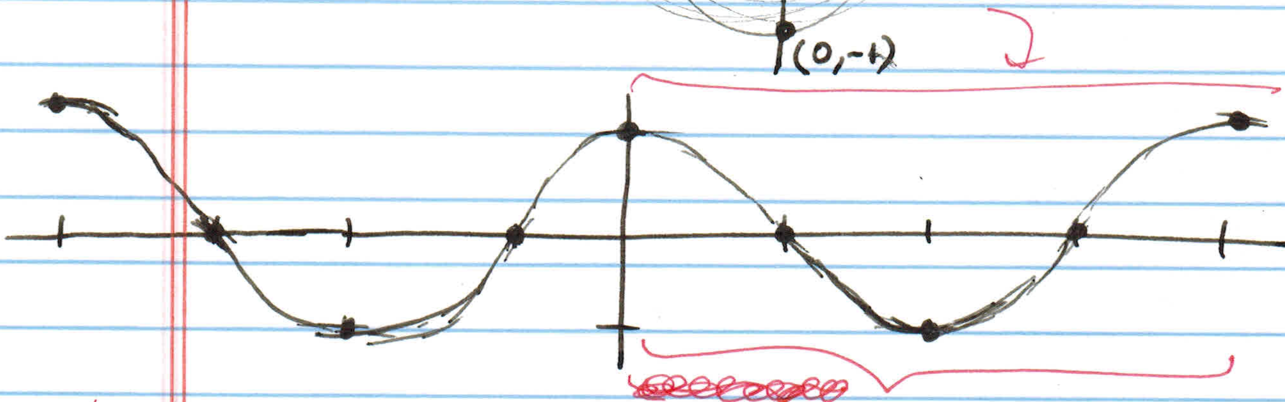
> The amplitude of $\sin(x)$ is 1.

QUICK #5

We can graph cos(t) the same way

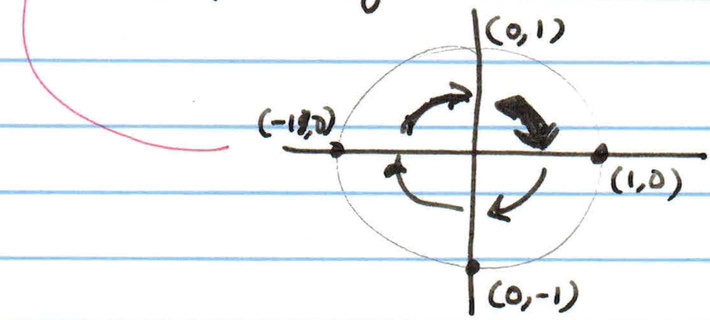
Remember: $\cos(t) = x$-value on unit circle





this is one period of cos(t)

for negative ∡'s



Domain of cosine = $(-\infty, \infty)$

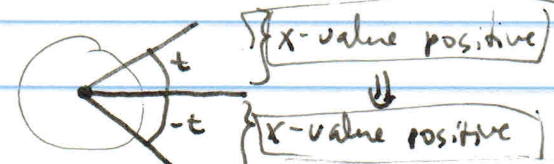
Range = $[-1, 1]$

Amplitude = 1

Period = 2π $\quad \left(\cos(t+2\pi) = \cos(t) \right)$

and

$\cos(-t) = \cos(t)$



The Period of

$$f(\boxed{x}) = \sin(3\boxed{x})$$

is $\frac{2\pi}{3}$

why is this true?

Check Algebraically:

$$f\left(\boxed{x + \frac{2\pi}{3}}\right) = \sin\left(3 \cdot \left(\boxed{x + \frac{2\pi}{3}}\right)\right)$$

$$= \sin(3x + 2\pi)$$

$$= \sin(3x)$$

So

$$f\left(x + \frac{2\pi}{3}\right) = f(x)$$

this means that $f(x) = \sin(3x)$ has period $\frac{2\pi}{3}$

some constant #

In General

$$f(x) = \sin(B \cdot x)$$

has period $\dfrac{2\pi}{B}$

To check this

$$f\left(\boxed{x + \tfrac{2\pi}{B}}\right) = \sin\left(B \cdot \boxed{\left(x + \tfrac{2\pi}{B}\right)}\right)$$

$$= \sin(Bx + 2\pi)$$

$$= \sin(Bx)$$

$$f\left(x + \tfrac{2\pi}{B}\right) = f(x)$$

To Graph $f(x) = \sin(3x)$,

I can try cutting $\frac{2\pi}{3}$ into 4 equal parts

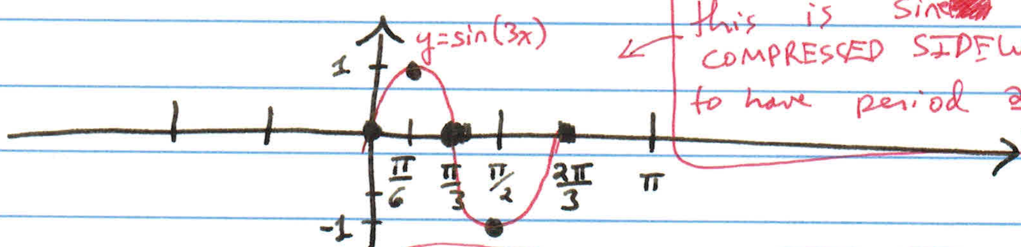
When graphing f w/ period 2π

$0 \qquad \frac{\pi}{2} \qquad \pi \qquad \frac{3\pi}{2} \qquad 2\pi$

When graphing f w/ period $\frac{2\pi}{3}$:

$0 \qquad \frac{\pi}{2}\cdot\frac{1}{3} \qquad \pi\cdot\frac{1}{3} \qquad \frac{3\pi}{2}\cdot\frac{1}{3} \qquad 2\pi\cdot\frac{1}{3}$

	x	$f(x) = \sin(3x)$
$0\cdot\frac{1}{3} =$	0	$f(0) = \sin(3\cdot 0) = \sin(0) = 0$
$\frac{\pi}{2}\cdot\frac{1}{3} =$	$\frac{\pi}{6}$	$f\left(\frac{\pi}{6}\right) = \sin\left(3\cdot\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$
$\pi\cdot\frac{1}{3} =$	$\frac{\pi}{3}$	$f\left(\frac{\pi}{3}\right) = \sin\left(3\cdot\frac{\pi}{3}\right) = \sin(\pi) = 0$
$\frac{3\pi}{2}\cdot\frac{1}{3} =$	$\frac{\pi}{2}$	$f\left(\frac{\pi}{2}\right) = \sin\left(3\cdot\frac{\pi}{2}\right) = \sin\left(\frac{3\pi}{2}\right) = -1$
$2\pi\cdot\frac{1}{3} =$	$\frac{2\pi}{3}$	$f\left(\frac{2\pi}{3}\right) = \sin\left(3\cdot\frac{2\pi}{3}\right) = \sin(2\pi) = 0$



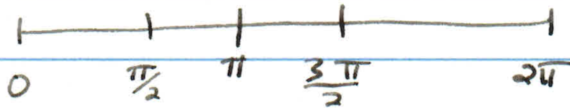
this is sine COMPRESSED SIDEWAYS to have period $\frac{2\pi}{3}$

Visually, you can see why $\sin(3x)$ has period $\frac{2\pi}{3}$

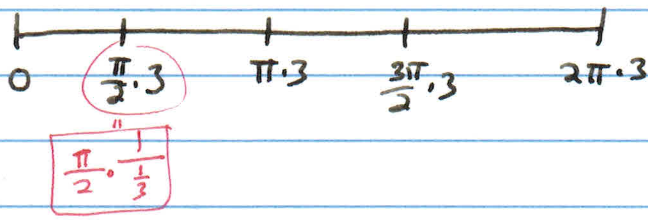
To Graph $f(x) = \sin\left(\frac{x}{3}\right) = \sin\left(\frac{1}{3} \cdot x\right)$

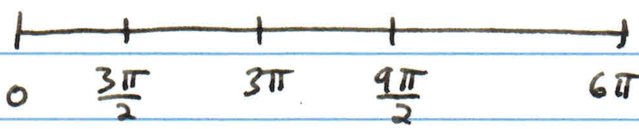
the Period of $f(x)$ is $\dfrac{2\pi}{\frac{1}{3}} = 6\pi$

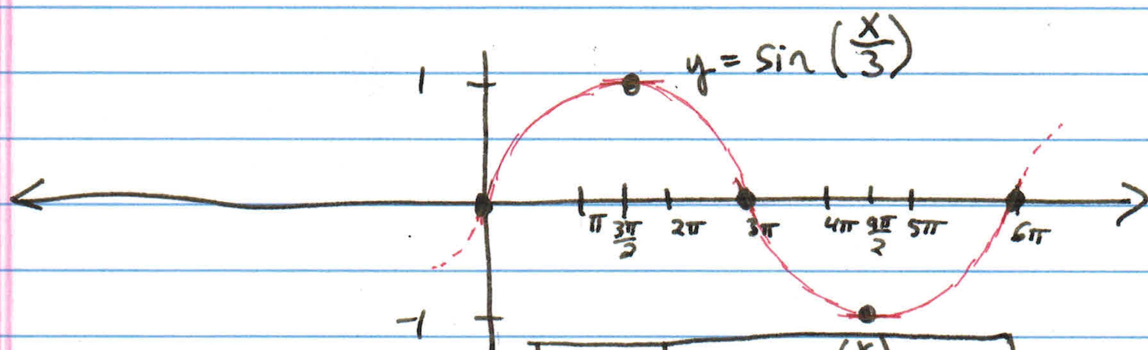
→ stretch the period of sine



to a period of 6π







x	$f(x) = \sin\left(\frac{x}{3}\right)$
0	$f(0) = \sin(0) = 0$
$\frac{3\pi}{2}$	$f\left(\frac{3\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$
3π	$f(3\pi) = \sin(\pi) = 0$
$\frac{9\pi}{2}$	$f\left(\frac{9\pi}{2}\right) = \sin\left(\frac{3\pi}{2}\right) = -1$
6π	$f(6\pi) = \sin(2\pi) = 0$

Name: ______________________________ Section: ______________________________

Horizontal Compressing/Stretching

multiply $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

By $\frac{1}{B} = \frac{1}{2}$

Graph: $h(x) = \sin(2x)$

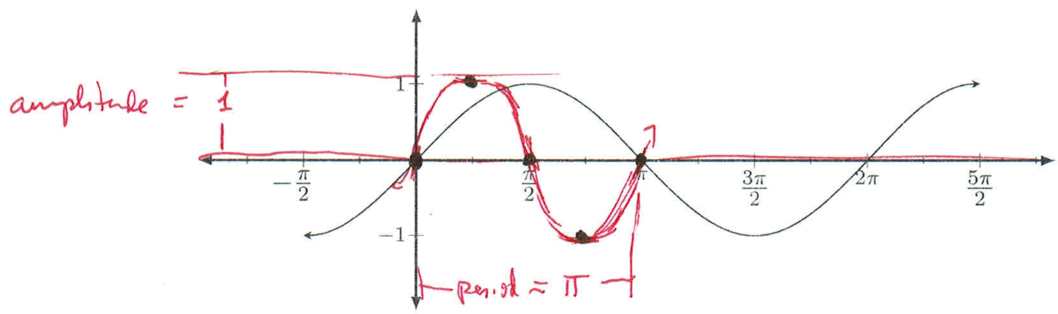
$0 \le 2x \le 2\pi$

$\Leftrightarrow$

$0 \le x \le \pi$

$\Rightarrow \boxed{\text{Period} = \pi}$

x	$h(x)$
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
$\frac{2\pi}{2}=\pi$	0



Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Amplitude: 1

Period: π

Graph: $h(x) = \sin(\frac{x}{2})$

multiply $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

By $\frac{1}{B} = \frac{1}{\frac{1}{2}} = 2$

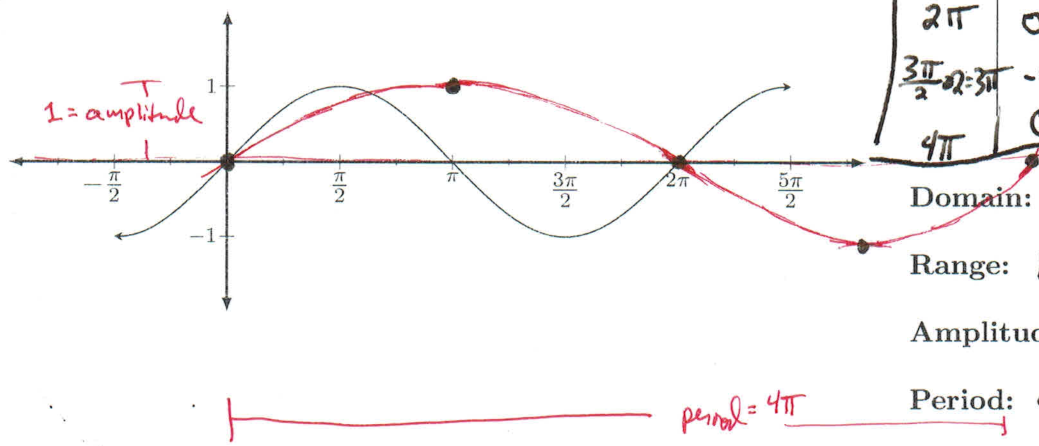
$0 \le \frac{x}{2} \le 2\pi$

$\Leftrightarrow$

$0 \le x \le 4\pi$

$\Rightarrow \boxed{\text{period} = 4\pi}$

x	$h(x)$
0	0
$\frac{\pi}{2} \cdot 2 = \pi$	1
2π	0
$\frac{3\pi}{2} \cdot 2 = 3\pi$	-1
4π	0



Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

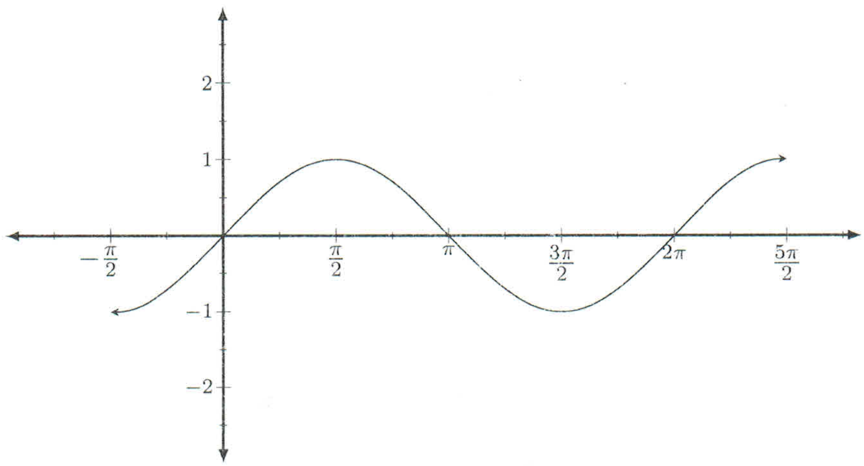
Amplitude: 1

Period: 4π

Name: ______________________ Section: ______________________

Combining operations: Part I

Graph: $h(x) = \sin(2x + \pi) =$ $\sin\left(2\left(x + \frac{\pi}{2}\right)\right)$



Helper Graphs

1. $\sin(x)$ – sin x
2. $\sin(2x)$ – compressed to period π
3. $\sin\left(2\left(x + \frac{\pi}{2}\right)\right)$ – moved left $\frac{\pi}{2}$

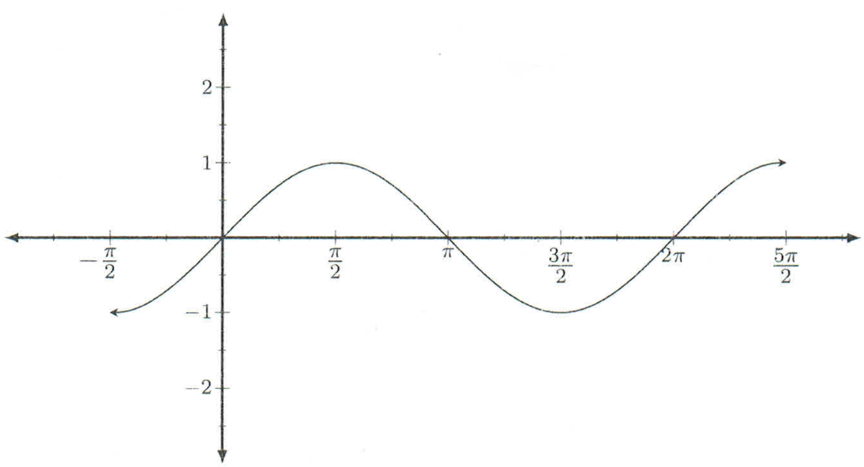
Domain:

Range:

Amplitude:

Period:

Graph: $h(x) = \sin(\frac{x}{2} + \frac{\pi}{4}) =$ $\sin\left(\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right)$



Helper Graphs

1. $\sin(x)$ – sin x
2. $\sin\left(\frac{1}{2}x\right)$ – stretched to have period 4π
3. $\sin\left(\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right)$ – moved left $\frac{\pi}{2}$

Domain:

Range:

Amplitude:

Period:

To Sketch

$$f(x) = A \cdot \sin(Bx - C) + E$$

Notice *

$$f(x) = A \cdot \sin\left(B\left(x - \frac{C}{B}\right)\right) + E$$

this is

① $\sin(Bx)$ ← period stretched/compressed to $\frac{2\pi}{B}$

② moved over $\frac{C}{B}$ to get

$$\sin\left(B\left(x - \frac{C}{B}\right)\right) = \sin(Bx - C)$$

③ stretched vertically by A to get

$$A \cdot \sin(Bx - C)$$ ↖ amplitude changed to $= A$

④ moved up/down by E to get

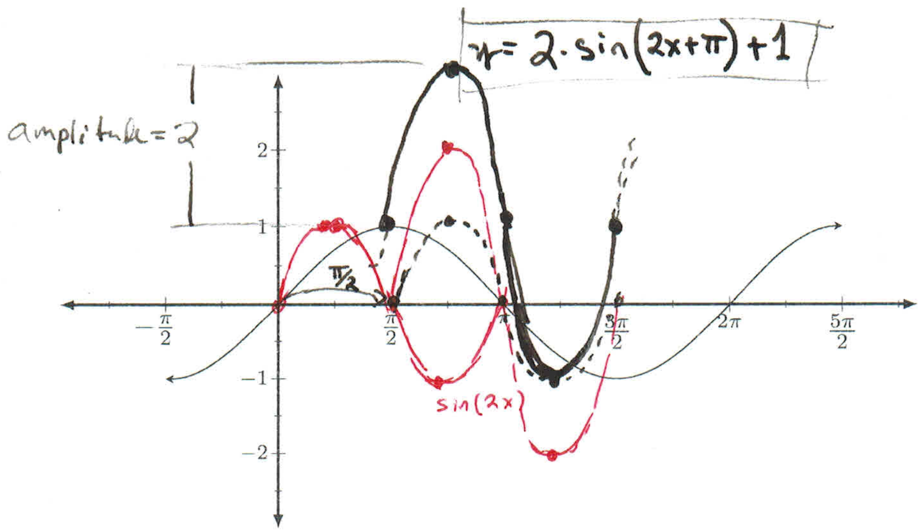
$$f(x) = A \cdot \sin\left(B\left(x - \frac{C}{B}\right)\right) + E$$

Name: ____________________ Section: ____________________

Combining operations: Part II

Graph: $h(x) = 2\sin(2x + \pi) + 1 = 2 \cdot \sin\left(2\left(x + \frac{\pi}{2}\right)\right) + 1$

A B C



Period = $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$

Helper Graphs

1. $\sin(2x)$
2. $\sin\left(2\left(x + \frac{\pi}{2}\right)\right)$
3. $2 \cdot \sin(2x + \pi)$
4. $2 \cdot \sin(2x + \pi) + 1$

Domain:

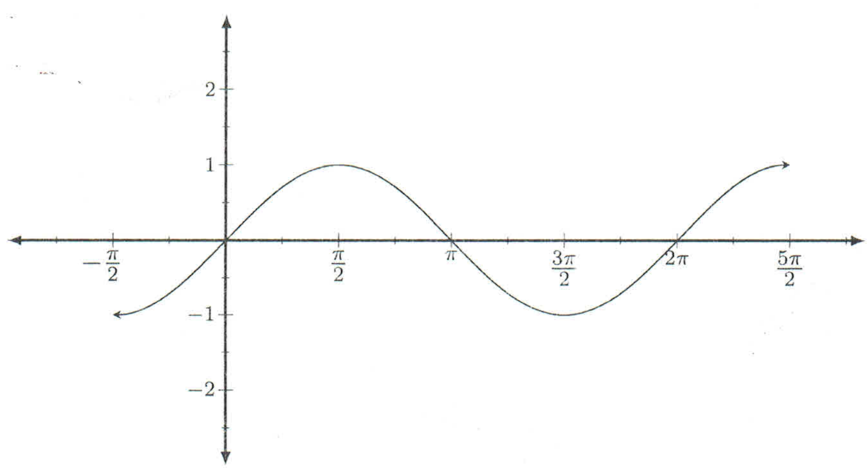
Range:

Amplitude: $A = 2$

Period: $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$

Graph: $h(x) = \frac{1}{2}\sin\left(\frac{x}{2} + \frac{\pi}{4}\right) - 1 = \frac{1}{2} \cdot \sin\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) - 1$

A B C



Helper Graphs

1. $\sin\left(\frac{1}{2}x\right)$
2. $\sin\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$
3. $\frac{1}{2}\sin\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$
4. $\frac{1}{2}\sin\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) - 1$

Domain:

Range:

Amplitude: leading coefficient $A = \frac{1}{2}$

Period:

$\frac{2\pi}{B} = \frac{2\pi \cdot 2}{\frac{1}{2} \cdot 2} = 4\pi$